ACTA DIDACTICA NAPOCENSIA
Volume 8, Number 1, 2015

# VISUAL REPRESENTATIONS IN MATHEMATICS TEACHING: AN EXPERIMENT WITH STUDENTS

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**Abstract:** General problem-solving skills are of central importance in school mathematics achievement. Word problems play an important role not just in mathematical education, but in general education as well. Meaningful learning and understanding are basic aspects of all kinds of learning and it is even more important in the case of learning mathematics. In order to efficiently enhance students' problem solving skills they should be assigned world problems which are new to them and to which they themselves have to find the steps to the solution, the algorithm. A number of researches, experiments and scientific papers in the didactics of mathematics prove that the role of visual representations in word problem solving is essential. Visual representation often helps in understanding a problem. Using visual representations leads to a better understanding and to improving special mathematical reasoning. The present paper presents an experiment involving university students who have been asked to solve a logical problem. There were three experimental groups. The aim was to find out whether problem solving is more efficient when using the traditional method (working on paper, with pencil) or when using a model.

**Key words:** mathematical competence, problem-solving skills, word problems, comprehension, representations, mathematical model.

## 1. Introduction

Mathematical competence encompasses the development and use of skills and abilities related to mathematical reasoning. The OECD PISA mentions the following eight components of mathematical competence. [15]

1. reasoning, making deductions
2. argument and elaborating a proof
3. communication
4. modelling
5. finding solutions
6. representation
7. using symbolic, formal and technical language and operations
8. using mathematical tools

The mathematical competence model based on factor and content analysis contains a detailed list of abilities and skills of mathematical competence [2], see Table 1 for details.

Table 1. Mathematical competence model [2, 6]

| | |
|---|---|
| Skills | counting, calculation, quantitative reasoning, estimation, measuring, measurement conversion, word problem solving |
| Reasoning skills | systematization, combinatorial reasoning, deductive and inductive reasoning, calculating probabilities, argumentation |
| Communication skills | relation terminology, comprehension, text interpretation, spatial visualization, visual-spatial representation, presentation |
| Acquisition skills | problem receptivity, problem representation, originality, creativity, problem solving, metacognition |
| Learning skills | concentration, part-whole awareness, memorization, problem solving speed |

*Received February 2015.*

Word problem solving as well as comprehension and text interpretation are present among the skills in the mathematical competence model. Problem solving is one of the acquisition skills.

Representation and presentation are also components of mathematical competence belonging to acquisition and communication skills respectively.

The *Mathematics Programmes of Study* effective in Romania does not present the objectives for word problem solving for school grades 1 to 8. There are no details concerning the steps or levels attaching them to grades or age groups.

The performance standards for the end of elementary education state as an objective "using arithmetic reasoning in solving word problems" as well as "solving problems that require three operations at most." [14]

As concerns secondary school only the $5^{th}$ grade curriculum contains as an objective "solving word problems with the help of equations and inequalities " When it comes to higher grades solving word problems is mentioned only in a general context, e.g. "solving practical problems using operations with real numbers." [14]

One can raise the question what about the different types of word problems which cannot be solved with equation, e.g. logical, combinatorial or other specific word problems which need more complex strategies.

Course books or workbooks do not contain any open problems.

## 2. Reading, word problems, representations

According to a present-day definition "word problems are real- life, practical problems in which the correlation between the known and unknown quantities are provided in the form of text, and their solutions need some kind of mathematical model." [12]

Word problems also play a crucial role in forming the concept of operations and indirectly in practicing operations. Working with word problems in elementary education creates a base for the ability to model more complex, practical problems. Word problems also help in developing comprehension, judging, memorization and self-check abilities.

To an efficient development of problem-solving skills it is recommended to use more new word problems to which students have to find the steps leading to the solution, the algorithm.

In all stages of education we must place great emphasis on word problems, on their correct interpretation, understanding, on observing the steps in problem solving, possible representations, interpreting results in terms of real world situations because word problems play an important role in developing comprehension.

How do we solve word problems? There are different models for this. Pólya's model develops reasoning and divides problem solving into four steps. [10]

I. *Understand the problem!*

II. *Look for connections between data and the unknown! If you cannot find direct connections look for helping problems! Make a plan for solving the problem!*

III. *Carry out the plan!*

IV. *Check the solution!*

Methods for solving word problems: general methods, as well as particular methods such as representation, contrasting, hypotheses, backwards working, rule of three, method of balancing.

Numerous psychological studies prove that the representation of mathematical objects and pictorial representations play an important role in the learning process. Illustrations facilitate a better understanding of concepts or of a problem, they help develop mathematical reasoning.

The essence lies in representing the data of the problem, the unknown and the relations between them, and using the representation to analyze and solve the problem. One can use sketches, plane figures, segments, symbols and conventional signs or letters. Representation is important since it contributes to a better understanding and memorization of the problem. [11]

[7] analyzed the structure of internal mathematical representations and found that the imagistic system (nonverbal configurations of objects, relations and transformations, including visual imagery and spatial representations) receives much less attention from educators than other systems of mathematical representations. It is clear from Goldin & Kaput 's argumentation that the ability to visualize data and their relations in a mathematical problem may contribute to mathematics problem solving.

Hegarty & Kozhevnikov [8] have found that "some visual- spatial representations promote problem-solving success".

Drawings used for solving word problems can be categorized based on [9] as schematic and pictorial.

Schematic drawings modelling mathematic quantities and relations play an important role in the cognitive processes involved in mathematical problem solving. "Mental representations generated in the process of word problem solving can be introduced as early as elementary education. Students who are made aware of mental representations built on visual imagery perform better, and change their convictions related to mathematics." [3]

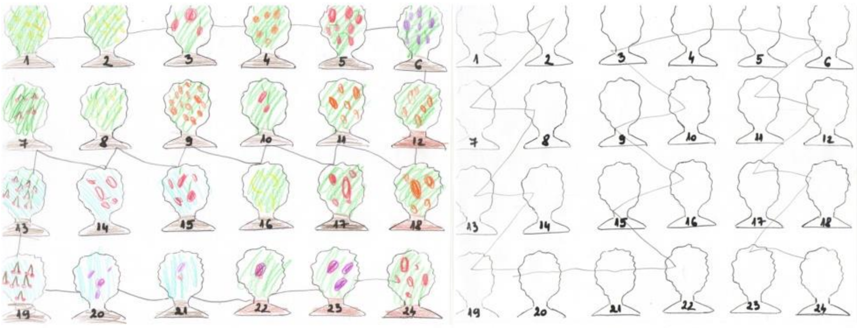

Figure 1. 2nd grade students solve problems with the help of representations (pictorial and schematic drawings)

Students have to be made aware of using visual representations. This should be done through practice, with a lot of patience." Using concrete and iconic representations is necessary not only for the so called slow students or elementary students. These representations are important for all students and are useful throughout the entire learning process". [13]

"One mode of representation does not suffice for the conditions and requirements of solving a problem or managing a situation. Most often multiple representation is asked for. A parallel engagement of different modes of representation and the connection between these yields a more efficient activity. Mathematical power lies in the properties separate from representations and the connection between representations". [5]

In [4] is presented a research carried out on teacher training graduates. In this we measured the extent to which future teachers could be sensitized to using representations when solving word problems, thus facilitating a better understanding of the problem.

The present research was carried out on three groups who had to solve an unusual logical problem. The aim was to find out whether problem solving is more efficient when using the traditional method (working on paper, with pencil) or when using a model.

## The research

*The aim of the research:* asking the students to solve the Einstein-type problem, we wanted to measure the level of their independent thinking, their problem solving skills, and their practical application of knowledge. We hypothesized that representations (using models) might prove helpful in solving the problems.

57 Economics freshmen (specializing in Bank and Finances, Management or Tourism) at Partium Christian University have been asked to solve the Einstein-type problem in three different ways.

*The structure of the research*

I. group: students worked out the logical steps to the solution using the traditional method. The exercise was presented on paper and they were allowed to use only paper and a writing tool.
II. group: students solved the problem and the operations on the model using cards.
III. group: students carried out the logical steps on a computer, they received a representation of the problem. They had to run a program made in advance (they made operations using virtual cards)

*The problem used for the experiment*

Einstein's problem:

1. There are 5 houses in 5 different colours.

2. In each house lives a person of a different nationality.

3. These 5 owners drink a certain beverage, smoke a certain brand of cigar, and keep a certain pet.

4. No owners have the same pet, smoke the same brand of cigar, or drink the same beverage.

The question is: **Who keeps fish?** Hints:

1. The Brit lives in a red house.
2. The Swede keeps dogs as pets.
3. The Dane drinks tea.
4. The green house is on the left of the white house (next to it).
5. The green house owner drinks coffee.
6. The person who smokes Pall Mall rears birds.
7. The owner of the yellow house smokes Dunhill.
8. The man living in the house right in the center drinks milk.
9. The Norwegian lives in the first house.
10. The man who smokes blend lives next to the one who keeps cats.
11. The man who keeps horses lives next to the man who smokes Dunhill.
12. The owner who smokes Blue Master drinks beer.
13. The German smokes Prince.
14. The Norwegian lives next to the blue house.
15. The man who smokes blend has a neighbor who drinks water.

Einstein devised this problem a century ago. He said that 98% of the people in the world couldn't solve it. Enjoy!

## The results of the survey

Table 2. below contains the number of students divided into groups. The first and second group consisted of 16 students per group, (i.e. 28,07% in each group of the total of students involved), the third group consisted of 25 students (43,85 %-of the total). Students have been assigned to groups randomly.

Table 2. The proportion of students involved in the research

| | |
|---|---|
| Students (total) | **57 (100%)** |
| **I. group:** traditional method (paper, pen) | **16 ( 28,07%)** |
| **II. group:** using model (cards) | **16 ( 28,07%)** |
| **III. group:** using a computer program | **25 (43,85 %)** |

Table 3. below contains the results of problem solving for all three groups. Students who worked with representations (group II and III ) achieved better results. They solved the problem in a proportion of 100% and 64% respectively. Students who used the traditional method, working on paper managed to solve the problem in a proportion of only 62,5%.

Table 3. Solutions by students involved in the research

| Students | Proportion of students who solved the problem | Avarage time (minutes) | Dispersion |
|---|---|---|---|
| I. group | 62,50 % | 35,1 | 11,50 |
| II. group | 100 % | 43,93 | 19,83 |
| III. group | 64% | 34,5 | 11,91 |
| Total | 73,68 % | 38,23 | 14,82 |

On the whole 73,68% of the students managed to solve the problem. The avarage time of those students from group I and III who managed to solve the problem is 35 minutes (rounded dispersion 12 minutes). Students from group II took a bit longer, almost 44 minutes (dispersion almost 20 minutes), however everyone in this group solved the problem, thus it reflects the avarage time for the whole group.

## Conclusions

There is a strong need for the use of concrete and visual representations in the teaching of mathematics in schools. According to Bruner in order for a student to understand the learning material he/she needs to process it in an intuitive way at an early stage and later revise it in conformity with his/her level of development (*spiral* curriculum). [1]

What we learn from the research is that even a difficult problem (of which Einstein himself said that 98% of the people cannot solve it) can be solved if presented in a way which facilitates understanding. (It might have seemed to be a very difficult problem a hundred years ago, however today it is no longer so!)

We hypothesized that representations (models) will help students solve the problem. The hypothesis turned out to be correct as students who worked with representations (II and III group) achieved better results. They solved the problem in a proportion of 100% and 64% respectively, while students from group I who used the traditional method, working on paper managed to solve the problem in a proportion of only 62,5%. What we found surprising was that students from group II, who used the cards, were more persevering, their motivation lasted longer, they didn't give up. They often restarted the exercise, nobody felt frustrated (the only thing that bothered them was to see that their colleagues had already finished and they had not). We did not set time limits for solving the problem. Students could work for as long as their patience lasted.

To our surprise students from group III, who carried out operation using virtual cards, achieved similar results to those in group I. They started out with great enthusiasm and motivation but in the end 36% gave up, they did not finish solving the problem similarly to group I (37,5% gave up) their interest decreased as they started to get tired.

After carrying out the research we talked to each student asking them how they had felt what positive or negative experiences they gained. There were people in group II who had restarted the problem five or three times. Two students had asked for paper and solved the problem using tables and then placed the cards accordingly. They said it was easier for them this way. (They had used a different representation).

Many students from the same group said that they had devised strategies, they had noticed that cards can be paired “first I placed the ones I was sure of”, “I realized that if I make groups I have to place those out at once” “I worked in pairs” “I am a visual type so this method was easier for me” “I solved something similar when I was in 6th grade”.

Following the research some questions remain unanswered. For example:

1. How could we implement as many visual representations as possible in the teaching-learning process in higher education?

2. Students from group III, who used computers, received a representation of the problem; they had to use a programme and worked with operations. Why haven’t they also achieved better?

We are planning to continue the research drawing comparisons between different approaches using other problems.

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